Product Differentiation, Semiorder Preferences and Price-Oriented Buyers

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Abstract

We introduce behavioral considerations in the demand side of a product differentiation model. In particular, we assume a population of buyers with semiorder preferences who are heterogeneous in two dimensions: their bliss points and their sensitivity towards product differences. This allows to endogenize the segmentation of buyers into two groups: price-oriented buyers and non-price-oriented buyers, which affects sellers' incentives to differentiate their products. While we derive conclusions for the study of product differentiation, the model is tractable and intuitive and can be applied in several contexts.

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1 Introduction

Product differentiation is one of the most studied strategies to relax price competition. It relies on the intuitive idea that consumers are willing to pay a higher price for a variety of product that is more in line with their tastes. However, product differentiation admits the existence of differences between goods that are negligible for some buyers. Let us consider two companies that make cookies and assume that these cookies only differ in their chocolate-to-almond composition, as well as in price. Chocolate lovers may prefer cookies with a larger amount of chocolate, while almond lovers may prefer a variety that contains a higher proportion of almonds. If the cookies from both companies contain an identical combination of the two ingredients, then price will be the only difference between the two products and consequently, the only determining factor in the buyers' decision. Now, let us imagine that one brand of cookies contains one milligram more

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of chocolate than the other. In all likelihood, most buyers will not care about (or even notice) the difference and, therefore, price will remain the only variable determining their final decision. The same will probably be true for two milligrams, three milligrams and other small variations. Nonetheless, more buyers may progressively take into account the cookies' ingredients into their purchasing decision, as long as the proportion of chocolate in one of the varieties increases. Consequently, price will no longer be the sole attribute that determines their purchasing decision.

Two features of the above example are relevant for the study of product differentiation. First, differences in varieties might be negligible, i.e. buyers will only care about the chocolate-almond composition of the cookies if they deem the existing difference between the two alternatives as relevant. This non-rational aspect of buyer behaviour is the famous notion highlighted in Luce's (1956) semiorders and further explored by Tversky (1969).[1] Second, individuals might be heterogeneous in the level of product differentiation that they consider to be relevant. For instance, a diabetic buyer might be more selective regarding the cookies' ingredients than a non-diabetic one; or a very hungry buyer might be less concerned with the cookies' ingredients than somebody who is non-hungry and only wants to enjoy the pleasure of eating a cookie. This heterogeneity implies that a larger share of buyers will take into account the chocolate-almond composition of the cookies as long as the difference between the two alternatives increases. Similarly to the previous example, people may care differently about product variety in several contexts. For instance, only a few people may be interested in differences in the design of small appliances such as toasters. Youngsters tend to be more aware of differences in technological products than older generations. Some people do not care about having an iOS or an Android operative system in their smartphones, while others see them as something completely different. Passengers with a high opportunity cost of a minute, e.g. business executives, may be more concerned about flight schedules than passengers with a more flexible agenda. Car lovers pay attention to differences in automobiles that generally go unnoticed by many drivers.In most of these contexts, consumers may not only differ in their favourite type of product but also in their concern towards product differentiation. Only those consumers who care about the existing differences between two products will be willing to pay a higher price for a particular variety. Moreover, we may expect the amount of buyers who do not care on product differences to be larger, the more similar are the two products. In the present paper, we explore how individuals' heterogeneity

[1] Semiorder preferences are not rational because violate transitivity of the indifference relationship. If an individual is indifferent when two cookies differ in 1 mg of chocolate but not when they differ in 2 mg, then she will be indifferent between two cookies with 5 and 6 chocolate mg respectively and between two cookies of 6 and 7 chocolate mg but not between two cookies containing 5 and 7 chocolate mg, respectively.

in their concern towards product differences affects firms' optimal strategies regarding product differentiation.

To capture these intuitive ideas, we introduce semiorder preferences (Luce, 1956; Tversky, 1969) in a Hotelling (1929) duopoly model. In particular, we assume that when the products of two firms are similar enough, some buyers may not care about product variety.[2] These buyers will be price-oriented because, given that they do not value the characteristics of the two products differently, they will buy the cheapest one.[3] In a standard Hotelling model, price-oriented decisions only occur in the case of identical products. However, as recognized by the marketing discipline, the existence of price-oriented buyers is a general phenomena that can also occur in the presence of non-identical products; see Baye et al. (2004), Bolton and Myers (2003), Kotler et al. (2014) or Murakata and Matsuo (2010), among others.[4] We generalize the Hotelling model to allow the possibility that slightly differentiated but not necessarily identical products induce some buyers to be price-oriented. Importantly, we allow buyers to be heterogeneous on the degree of product differentiation that induces them to be price-oriented. Consequently, the proportion of price-oriented buyers will be endogenous and will decrease in the degree of product differentiation. In other words, the amount of price-motivated buyers will be larger, the more similar are the two products. It is well known that one of the main determinants of demand price elasticity is the existence of close substitutes. In our setup, a reduction in product differentiation increases the number of buyers for whom both goods are perfect substitutes (i.e., price-oriented buyers' demand has infinite price-elasticity) and reduces the amount of buyers for which the two goods are imperfect substitutes.

In our model, price-oriented buyers have incentives to postpone their purchasing to a sales period.[5] The opposite is not true for non-price-oriented buyers, which generates an endogenous market segmentation. Price competition is à la Bertrand in the price-oriented segment, and hence, rents are fully dissipated when the two sellers have equal marginal costs. Consequently, sellers experience two different effects from drawing close to each other. On the one hand, by coming closer to their opponent, sellers capture a larger share of non-price-oriented buyers' demand. On the other hand, by getting closer to the other seller, they increase the total share of price-oriented buyers, from whom no

[2]The idea of similarity has also been studied in the context of decision-making under risk, see Rubinstein (1988).

[3]By juxtaposition, we call non-price-oriented buyers those who care about product differences. However, their decision will be motivated both by price and product variety.

[4]More generally, we can talk about heterogeneity in individual price elasticity; see Bijmolt et al. (2005) or Hoch et al. (1995).

[5]Our model substantially differs from the seminal sales models by Salop and Stiglitz (1982) or Varian (1980). An important difference, among others, is that in their cases, informed buyers are more prone to pay a lower price, while in our case, this happens for the price-oriented buyers (i.e., uninformed about variety but informed about price).

rents are obtained. If the rate of buyers who become price-oriented as the two sellers get close to each other is high enough, the latter effect dominates. Consequently, sellers have further incentives to differentiate compared with a standard Hotelling model. This reduces sellers' incentives to locate at the center, and compromises Hotelling's (1929) principle of minimal differentiation.[6] Moreover equilibrium varieties can emerge out of the region in which undercutting profits arise; see d'Aspremont et al. (1979). Consequently, subgame perfect equilibria with pure strategy equilibrium prices can exist. This contrasts with the well-known nonexistence of the pure strategy subgame perfect equilibrium of the Hotelling duopoly game with linear transportation cost. Under homogeneous marginal costs, in equilibrium, both sellers choose symmetric and sufficiently (but not necessarily maximally) differentiated varieties.

We also study the case in which one seller has a lower marginal cost. Differently from the homogeneous costs case, the rents of the cost-advantaged seller are not fully dissipated in the sales period. Consequently, the cost-advantaged seller is less harmed by the presence of price-oriented buyers and is willing to choose a more central variety than the disadvantaged seller. The favorable position in the product differentiation space enjoyed by the lower-cost seller induces her to fix a higher price than her competitor. Thus, counterintuitive equilibria may emerge in which the lower-cost firm sells the product at a higher price. The profit-maximizing mark-up rule studied in textbook models of monopolistic competition leads to a similar result if the demand of the low-cost firm is sufficiently less elastic than that of the high-cost firm. However, no previous research has explained why a lower-cost seller may have a less elastic demand. A nice aspect of our model is that we provide an endogenous explanation for it. The lower-cost seller is able to choose a more central variety with a less elastic demand because she is less reluctant to compete à la Bertrand for the segment of price-oriented buyers.

Tversky (1969) proposed semiorder lexicographic preferences to represent decision-making in the presence of multiattribute alternatives.[7] Fishburn (1974) describes this criteria as lexicographic preferences "disregarding" small differences. Alexis et al. (1968), Korhonen et al. (1990), Russ (1972) and Tversky (1969) find favorable evidence of lexicographic semiorder procedures in different experimental contexts. Buyers' choice in a Hotelling model can be considered a multiple attribute choice. Products differ in two attributes: price and variety. Since the seminal work of Hotelling, price-variety substitutability has been assumed to determine buyers' choices. However, the assumption of attribute substitutability has been often questioned in multiattribute decision making,

[6]Despite the mathematical mistake in the original Hotelling's article (d'Aspremont et al., 1979), several works have demonstrated the validity of the principle, see Irmen and Thisse (1998) and Xefteris (2013).

[7]Foerster (1979) refer to them as weak lexicographic preferences.

see Korhonen et al. (1990) and Tversky (1969). We relax the substitutability assumption by introducing a weaker version of the semiorder lexicographic preferences in a Hotelling model. As Tversky (1969, p. 40) recognizes, his proposal *"is based on a noncompensatory principle that is likely to be too restrictive in many contexts"*.[8] To overcome this limitation, Luce (1978) proposed a variant of lexicographic semiorders that relaxes Tversky's non-compensatory principle. In particular, it admits the possibility of substitutability if indifference arises in the dominant attribute. Despite this difference, Luce's proposal maintains many of the properties of lexicographic semiorders. In our paper, we adapt Luce's (1978) proposal to a Hotelling duopoly model. To the best of our knowledge, this is the first paper introducing behavioural considerations in the demand side of a spatial competition model. Previous works have studied the implications of introducing behavioural considerations in the supply side of spatial competition models (Balvers and Szerb, 1996; Barreda-Tarrazona et al., 2011).

Finally, we would like to emphasize that the contribution of the present article is not restricted to the study of consumer research and product differentiation. The endogenous segmentation of individuals based on the behavioral notion of semiorder preferences can be applied to other contexts. For instance, studies in Political Science often take into account models of spatial competition (Downsian models) in which our proposal can also be applied. In an ongoing research, Balart et al. (2017) exploits this idea in context of electoral competiton to explain some observed trends in the US politics.

In section 2, we present the model. We solve the different stages of the game in sections 3 to 5. Finally, we conclude in section 6.

2 The Model

2.1 Sellers

There are two sellers, A and B. Seller $I \in \{A, B\}$ can distinguish horizontally by locating their product in the continuum $i \in [0, 1]$. We assume, without loss of generality, that $a \leq b$. Sellers also have to choose the price of the good, $p_I \in \mathbb{R}_+$. As usual in spatial competition models, we assume that product type is chosen before price. After fixing the price, sellers can also apply a discount, which we denote by $d_I \in [0, p_I]$.

The marginal cost of producing the good of seller I is constant and equal to $c_I \in \mathbb{R}_+$. We assume without loss of generality that $0 = c_A \leq c_B$.

Sellers' profits can be written as $\Pi_I = p_I D_I - d_I \theta_I - c_I D_I$, where $D_I \in [0, 1]$ is the

[8] Mariotti and Manzini (2012) proposed sequential rationalizability of choice as an alternative to relax this restriction.

total demand served by seller I and $\theta_I \in [0, D_I]$ are the units sold under a discount.

2.2 Buyers

There is a population of buyers of measure 1. Each buyer purchases one unit of the good, either from seller A or seller B. Buyers are heterogeneous in two dimensions. First, they are heterogeneous regarding their favourite product, which is represented by $j \in [0, 1]$. Second, they are heterogeneous in their concern about product differences. We denote by ϕ the minimal distance between the two products that a buyer considers to be relevant. If the distance between the two products is less than ϕ, then the buyer sees the two products as equally valuable. That is, at equal prices, the buyer will be indifferent between two products with a degree of differentiation lower than ϕ. This corresponds to the notion of semiorder preferences first introduced by Luce (1956) and further explored by Tversky (1969). We call individuals with $\phi \geq b - a$ price-oriented buyers because price is the unique variable that they take into account for their purchasing decision. Otherwise, the buyer is non-price-oriented and chooses the good that presents the more attractive price-variety combination. Considering the cookies example in the introduction, a non-hungry buyer may have a small ϕ value, a somewhat hungry buyer may have a larger ϕ value and a very hungry buyer would have a very large ϕ value. We can also consider a chromatic example in which product varities only differ in their color; a colour-concerned buyer would have a small ϕ value, a colour-blind buyer would have a large ϕ value and a blind buyer would have $\phi = 1$, that is under price equality, he would be indifferent between the two goods even if the colour differentiation is maximal.

Although j is irrelevant for price-oriented buyers, all individuals are identified by the pair (j, ϕ). Note that for a buyer with a given value of ϕ being price-oriented or not exclusively depends on the existing degree of product differentiation; hence, the proportion of price-oriented buyers will be endogenously determined in our model. This behaviour can be represented by a variant of Tversky's (1969) lexicographic semiorder preferences. In particular, we adapt the semiorder lexicographic structure proposed by Luce (1978) to a spatial model of product differentiation:

$$U_{j,\phi}(I) = (M- \mid j - i \mid)\Upsilon(\phi < b - a) + m[1 - \Upsilon(\phi < b - a)] - \tilde{p}_I \qquad (1)$$

where $\tilde{p}_i = \{p_i - d_i, p_i\}$ depending on whether the buyer receives a discount and $\Upsilon(\phi < b - a)$ is an indicator function. We assume that the reservation price of any buyer is higher when he is non-price-oriented, that is, $M > m + 1$, and that both reservation prices are sufficiently high to guarantee that the market is covered. The higher reservation price enjoyed by non-price-oriented buyers can be understood in terms of hedonic shopping value; see, among others, Babin et al. (1994), Bloch and Bruce (1984), Childers et al. (2002) or Hirschman (1984). Only buyers who value the existing differences between the two products obtain hedonic value from the action of shopping.

From (1) it arises that buyers with a value of $\phi < b - a$ buy from seller A if and only if

$$p_A + \mid j - a \mid \leq p_B + \mid j - b \mid$$

These agents behave as in a standard Hotelling model. In contrast, also from (1), we see that buyers for which $\phi \geq b - a$ buy the cheapest product. Note that buyers' behaviour in a standard Hotelling model is a particular case of the proposed one with $\phi = 0$ for any buyer. Instead, we assume that ϕ may change across individuals.

Individuals' bliss points and concern towards product differences follow continuous cumulative distribution functions $G(j)$ and $F(\phi)$, respectively. We assume that these two are independent, i.e., $G(j|\phi) = G(j)$ and $F(\phi|j) = F(\phi)$. As usual in the Hotelling model, we assume that buyers' bliss points are uniformly distributed, that is, $G(j) \sim U(0,1)$, but we do not assume a specific functional form for $F(\phi)$. Then, we can represent the aggregate demand to each seller conditional on a, b, p_A, p_B, d_A, d_B as:

$$D_I(a, b, p_A, p_B, d_A, d_B) = F(b-a)D_I^{NPO} + [1 - F(b-a)]D_I^{PO}$$

where D_I^{NPO} and D_I^{PO} represent the non-price-oriented and price-oriented buyers' demand to seller I. Since $F(.)$ is a cumulative probability function, the share of price-oriented buyers is increasing in $b - a$.

Buyers can also choose whether to purchase the product at an initial period or to wait for sales. Buyers discount the future at a rate $\delta \in [0,1]$. Hence, in case of waiting for the discount, the payoff is $\delta U_{j,\phi}(I)$. We assume that $\delta \in [\frac{m-p_I}{m-p_I+d_I}, \frac{M-1-p_I}{M-1-p_I+d_I}]$, for $I = A, B$. This assumption guarantees that for any j, i, p_I, d_I:

$$M - \mid j - i \mid - p_I \geq \delta(M - \mid j - i \mid - p_I + d_I) \quad \text{and} \quad m - p_I \leq \delta(m - p_I + d_I)$$

Hence, only price-oriented buyers are willing to wait for the discount, i.e., $\theta_I = 1 - F(\phi)$ for $I = A, B$.[9] In other words, the sales period allows price discrimination between the two types of consumers. An alternative example of price discrimination between price-oriented and non-price-oriented buyers can be found in the airline industry, where flights are horizontally differentiated by their departing time. Flight tickets are typically cheaper

[9] The higher reservation price of non-price-oriented buyers, i.e., $M > m + 1$, guarantees that the assumed interval for the discount rate is non-empty for any M, m, P_I, d_I. Also, note that the assumed interval on δ expands to $(0,1)$, as $m \to p_I$ and $M \to \infty$, guaranteeing that the intersection of the intervals on δ for $I = A, B$ is also non-empty under the appropriate assumptions on m and M. In equilibrium, $M > m + 1$ is sufficient to guarantee non-emptiness of the intersection.

if they are bought in advance. Travelers who have greater time flexibility can buy their tickets in advance. In this case, early buyers are more price oriented and are generally charged lower prices.[10]

Price discrimination is necessary to have equilibrium prices in pure strategies, and it ensures the tractability of our model.[11] We have motivated price discrimination in terms of differences in purchasing time of price-oriented and non-price oriented buyers, however, we can consider alternative reasons for why discounts can be targeted to price-oriented buyers. As in Galeotti and Moraga-Gonzlez (2008), sellers might be able to price discriminate between the two groups of buyers, especially if they use different purchasing channels, such as online or in-store purchasing. Even if both types of buyers use the online channel, sellers might be increasingly able to identify their type of buyer by taking advantage of information technologies such as big data. As a report from The Boston Consulting Group (Haugen et al., 2002) highlights, "*Recent advances in information technology have greatly sharpened the pricing tool, permitting ever finer consumer segmentation. The technology has done this in two ways. It has dramatically expanded the breadth and depth of information about consumers that companies can gather and process, and it has made possible the instantaneous delivery of customized pricing offers to individual consumers.*" Indeed, the possibilities of price discrimination offered by new technologies is an increasingly studied topic; see Anderson et al. (2015) and Liu and Zhang (2006), among others.

2.3 Timing of the game

Given the previous considerations, the timing of the game is as follows:

- At $t = 1$, sellers design their product, $i \in [0, 1]$.
- At $t = 2$, sellers decide the price of their product, p_I, $I = A, B$
- At $t = 3$, non-price-oriented buyers $F(b-a)$ buy the product with a more attractive price-variety combination
- At $t = 4$, sellers decide their discount, $d_I \leq p_I$, $I = A, B$
- At $t = 5$, price-oriented buyers $1 - F(\phi)$ buy the cheapest product.

We look for the subgame perfect equilibrium solving the game by backward induction.

[10] In contrast to in our model, in the airline example the discount is enjoyed by early buyers.

[11] Without the possibility of price discrimination, the price stage has an equilibrium in mixed strategies with no closed-form solution for the supports. This makes the variety selection stage intractable. The assumption of price discrimination has an intuitive implication: sellers obtain lower rents from price-motivated buyers.

3 Discount selection

To make their decision, price-oriented buyers only take discounted prices into account. We can write the price-oriented demand to seller A as:

$$D_A^{PO}(p_A, p_B, d_A, d_B) = \begin{cases} 0 & \text{if } p_A - d_A > p_B - d_B \\ 1 & \text{if } p_A - d_A \leq p_B - d_B \end{cases}$$

The aggregate demand of price-oriented buyers to seller B can be written as $D_B^{PO}(p_A, p_B, d_A, d_B) = 1 - D_A^{PO}(p_A, p_B, d_A, d_B)$.[12] Seller I chooses its discount to maximize: $[1 - F(b-a)](p_I - c_I)D_I^{PO}$. Thus, competition for price-oriented buyers can be seen as Bertrand competition, in which sellers directly decide the discount rather than the price. Given that $c_A \leq c_B$, the unique Nash equilibrium of this subgame is:

$$d_A = p_A - c_B \qquad\qquad d_B = p_B - c_B$$

Consequently, the payoffs are $c_B - c_A$ and 0 for seller A and B, respectively.

4 Price Selection

Prior to the sales period, sellers have to price their good. As usual in the Hotelling model, non-price-oriented buyers purchase the good according to its price and variety; hence:

$$D_A^{NPO}(p_A, p_B, a, b) = \begin{cases} 0 & \text{if } b - a < p_A - p_B \\ \frac{a+b-p_A+p_B}{2} & \text{if } a - b \leq p_A - p_B \leq b - a \\ 1 & \text{if } p_A - p_B < a - b \end{cases}$$

and $D_B^{NPO}(p_A, p_B, a, b) = 1 - D_A^{NPO}(p_A, p_B, a, b)$.[13]

Given the above equilibrium in the discount subgame, the price selection problem can be written as:

$$\max_{p_I} F(b-a)D_I^{NPO}(p_I - c_I) + [1 - F(b-a)](c_B - c_I) \quad \text{for } I = A, B$$

Note that the top right term is always zero for seller B. Assuming that both firms have positive demands from non-price-oriented buyers (we check this below) and given that $c_A = 0$, the sellers' best responses are:

[12] For simplicity, we assume a tie-breaking rule in favour of the lower-cost seller to guarantee equilibrium existence in pure strategies. We can remove this simplifying assumption with no consequences for our results; see Blume (2003).

[13] As in d'Aspremont et al. (1979), we assume $D_A^{NPO}(p_A, p_B, a, b) = a$ whenever $a = b$ and $p_A = p_B$. When $a = b$, undercutting incentives arise under any tie-breaking rule, hence this assumption is irrelevant for our results.

$$p_A(p_B) = \frac{p_B + a + b}{2} \qquad p_B(p_A) = \begin{cases} \frac{2+c_B+p_A-a-b}{2} & \text{if } c_B \in [0, 2+p_A-(a+b)] \\ c_B & \text{otherwise} \end{cases}$$

Note that if $c_B > 2 + p_A - (a + b)$, seller B would make negative profits by fixing $p_B = \frac{2+c_B+p_A-a-b}{2} < c_B$. To avoid this situation, we restrict our analysis to sufficiently low levels of cost asymmetry, i.e., c_B is small enough.[14] A sufficient condition is $c_B \leq 2$.[15] Solving the system of equation for $c_B \leq 2$ we find:

$$p_A^* = \frac{2+a+b+c_B}{3} \qquad \text{and} \qquad p_B^* = \frac{4-a-b+2c_B}{3}$$

It is easy to verify that the second-order conditions are satisfied. However, these expressions can only be an equilibrium of the price subgame if both sellers have a strictly positive demand from non-price-oriented buyers, i.e., $a - b < p_A - p_B < b - a$. These are known as the non-undercutting conditions. As highlighted by D'Aspremont et al. (1979) an equilibrium in pure strategies of the overall game fails to exist whenever undercutting is profitable. Let us denote by p_I^U the undercutting price that assures seller I to serve all non-price-oriented buyers. Given the above prices p_A^*, p_B^*, seller I has no incentives to undercut her rival if and only if:

$\Pi_I(p_A^*, p_B^*, a, b) \geq \Pi_I(p_A^U, p_B^*, a, b)$ where $p_I^U = p_J^* - (b - a)$ for $I = A, B$ and $J \neq I$

For $c_B \leq 2$, the above condition for seller A can be written as:

$F(b-a)D_A^{NPO}(p_A^*, p_B^*, a, b)(p_A^*) + [1 - F(b-a)]c_B \geq (p_B^* - (b-a))F(b-a) + [1 - F(b-a)]c_B$

Cancelling out the terms $F(b - a)$ and $[1 - F(b - a)]c_B$, the above condition is identical to the non-undercutting condition in D'Aspremont et al. (1979), i.e.,

$$p_A^* D_A^{NPO}(p_A^*, p_B^*, a, b) \geq p_B^* - (b - a)$$

The same is obviously true for seller B. The following lemma arises immediately from D'Aspremont et al. (1979):

Lemma 1. *Undercutting is not profitable for any seller if and only if $a \neq b$ and*

$$\frac{(2+a+b+c_B)^2}{18} \geq \frac{2}{3}(2 + a - 2b + c_B) \quad \textit{and} \quad \frac{(4-a-b-c_B)^2}{18} \geq \frac{2}{3}(1 + 2a - b - c_B)$$

When these conditions are satisfied, the unique Nash equilibrium of the price subgame is $p_A^* = \frac{2+a+b+c_B}{3}$ and $p_B^* = \frac{4-a-b+2c_B}{3}$.

As shown by D'Aspremont et al. (1979) and Osborne and Pitchik (1987), any pair a, b has to satisfy the conditions in Lemma 1 for the existence of a subgame perfect equilibrium of the overall game in pure strategies.

[14] Otherwise, cycles arise compromising the existence of an equilibrium in pure strategies.

[15] A necessary and sufficient condition is $c_B \leq 4 - a - b$. However, we use the sufficient condition $c_B \leq 2$ to avoid making assumptions on the endogenous terms a, b.

5 Variety Selection

Using the equilibrium expressions of the continuation subgame, sellers choose their variety according to the following optimization problem:

$$\max_{a} \; F(b-a)\frac{(2+c_B+a+b)^2}{18} + [1-F(b-a)]c_B$$
$$\max_{b} \; F(b-a)\frac{(4-c_B-a-b)^2}{18}$$

Consequently, the associated first-order conditions are:

$$f(b-a)[-\frac{(2+c_B+a+b)^2}{18}] + 2F(b-a)\frac{2+c_B+a+b}{18} + f(b-a)c_B = 0 \quad (2)$$
$$\frac{f(b-a)}{2}(4-c_B-a-b) - F(b-a) = 0 \quad (3)$$

Sellers experience two different effects from getting closer to each other. On the one hand, by drawing closer to their opponent, sellers capture a larger share of the non-price-oriented buyers. On the other hand, by getting closer to the other seller, they reduce the total share of non-price-oriented buyers. The latter effect is certainly harmful for seller B, as she has zero profit from the price-oriented share of the demand. This reduces seller B's incentives to get closer to his opponent in comparison to a standard Hotelling model. For seller A, the incentives to get closer to the opponent can be higher or lower with respect to a Hotelling model. This will depend on the relative size of the profits selling to price-oriented buyers, i.e., c_B, and to non-price-oriented buyers, i.e., $\frac{(2+c_B+a+b)^2}{18}$.

In the symmetric case ($c_B = 0$), there is full rent dissipation for both sellers in the price-oriented share of the demand. Consequently, in that case, both sellers have lower incentives to approach each other than in a standard Hotelling model.

Lemma 2. *If $F(\phi)$ is log-concave, the first order conditions are necessary and sufficient for a maximum.*

Proof. To prove this lemma we show that the objective functions of the two sellers are strictly quasi-concave. Strict quasi-concavity guarantees that the first order conditions are necessary and sufficient for a unique maximiser.[16]

First of all, notice that the previous first-order conditions in (2) and (3) can be

[16]See for instance Theorem M.K.4 (p. 962) in Mas-Colell et al. (1995).

rewritten as:

$$\begin{aligned}\frac{\frac{\partial \Pi_A}{\partial a}}{f(b-a)} &= \frac{F(b-a)}{f(b-a)} - \frac{(2+c_B+a+b)^2 - 18c_B}{2(2+a+b)} = 0 \\ \frac{\frac{\partial \Pi_B}{\partial b}}{f(b-a)} &= \frac{4-c_B-a-b}{2} - \frac{F(b-a)}{f(b-a)} = 0\end{aligned} \tag{4}$$

We start by showing that $\frac{\partial(\frac{\frac{\partial \Pi_I}{\partial i}}{f(b-a)})}{\partial i} < 0$ for $I = A, B$. For seller A, $\frac{\partial(\frac{\frac{\partial \Pi_A}{\partial a}}{f(b-a)})}{\partial i} < 0$ can be written as: $\frac{\partial[\frac{F(b-a)}{f(b-a)}]}{\partial a} - \frac{\partial[\frac{(2+c_B+a+b)^2-18c_B}{2(2+c_B+a+b)}]}{\partial a} < 0$. Log-concavity implies that $(\frac{F(\phi)}{f(\phi)})' \geq 0$. Therefore, log-concavity guarantees that for seller A:

$$\frac{\partial[\frac{F(b-a)}{f(b-a)}]}{\partial a} = -(\frac{F(\phi)}{f(\phi)})' \leq 0 < \frac{\partial[\frac{(2+c_B+a+b)^2-18c_B}{2(2+c_B+a+b)}]}{\partial a} = \frac{1}{2} + \frac{9c_B}{(2+c_B+a+b)^2}$$

Which implies that $\frac{\partial(\frac{\frac{\partial \Pi_A}{\partial a}}{f(b-a)})}{\partial i} < 0$ is always true under log-concavity.

For seller B, $\frac{\partial(\frac{\frac{\partial \Pi_B}{\partial b}}{f(b-a)})}{\partial i} < 0$ can be written as: $\frac{\partial[\frac{4-c_B-a-b}{2}]}{\partial b} < \frac{\partial[\frac{F(b-a)}{f(b-a)}]}{\partial b}$. By log-concavity,

$$\frac{\partial[\frac{4-c_B-a-b}{2}]}{\partial b} = -\frac{1}{2} < 0 \leq \frac{\partial[\frac{F(b-a)}{f(b-a)}]}{\partial b} = (\frac{F(\phi)}{f(\phi)})'$$

Which guarantees that $\frac{\partial(\frac{\frac{\partial \Pi_B}{\partial b}}{f(b-a)})}{\partial i} < 0$.

Importantly, the above transformations in (4) do not modify the sign of $\frac{\partial \Pi_I}{\partial i}$, that is:

$$sign\{\frac{\partial \Pi_I}{\partial i}\} = sign\{\frac{\frac{\partial \Pi_I}{\partial i}}{f(b-a)}\}$$

Therefore, we conclude from the derivatives on $\frac{\frac{\partial \Pi_I}{\partial i}}{f(b-a)}$, $I = A, B$ that for any pair i and i' such that $\Pi_I(i) \leq \Pi_I(i')$, $\frac{\partial \Pi_I}{\partial i}(i)(i'-i) > 0$ which is a definition of strict quasi-concavity for a $\mathscr{C}^1$-function, see for instance De la Fuente (2000) [Theorem 3.7, p263]. Strict quasi-concavity is sufficient to guarantee that the first order conditions are necessary and sufficient for a maximum. □

Log-concavity is satisfied by many common distribution functions, such as Normal, Uniform, Chi, Chi-Square or Laplace, among many others; see Bagnoli and Bergstrom (2005). Our results will hold for any log-concave distribution function without the necessity of restricting to any specific distribution of ϕ.

Equalizing the first-order conditions of the two sellers, we obtain that an interior solution exists if and only if the following conditions are satisfied:

$$a + b = \frac{3\sqrt{4c_B+1} - 1 - 2c_B}{2} \quad \text{and} \quad \frac{F(b-a)}{f(b-a)} = \frac{9 - 3\sqrt{4c_B+1}}{4} \tag{5}$$

Log-concavity of $F(\phi)$ implies that there is at most one value of $b-a$ satisfying the second equality. We use ϕ^* to denote this value.

The next lemma provides an additional condition that must be fulfilled in any equilibrium.

Lemma 3. *In a pure strategy equilibrium with non-identical varities ($a \neq b$), $a \leq \frac{1}{2}$ and $b \geq \frac{1}{2}$.*

Proof. We proceed by contradiction. Consider an equilibrium such that $\hat{a} < \hat{b} < \frac{1}{2}$, and denote $\hat{\phi} = \hat{b} - \hat{a}$. We can show that there is always a profitable deviation for seller A. Consider $\tilde{a} = 2\hat{b} - \hat{a}$. The deviation does not modify profits arising from the price-oriented share of the demand, as it is independent of varieties. Moreover, after deviating to $\tilde{a}$, the total share of price-oriented and non-price-oriented buyers remains unchanged and equal to $1 - F(\hat{\phi})$, as $|\hat{b} - \tilde{a}| = \hat{b} - \hat{a}$. However, by deviating to $\tilde{a}$, seller A strictly increases her profit from non-price-oriented buyers, i.e., $\Pi_A^{NPO}(\hat{a}, \hat{b}) = \frac{(2+c_B+\hat{a}+\hat{b})^2}{18} < \frac{(4+c_B-(2\hat{b}-\hat{a})-\hat{b})^2}{18} = \Pi_A^{NPO}(\tilde{a}, \hat{b})$, for all $\hat{b} < \frac{1}{2}$. Consequently, $\tilde{a}$ constitutes a profitable deviation for seller A (everything remains the same as with $\hat{a}$ except for the higher profit arising from non-price-oriented buyers). We can proceed similarly to show that seller B can always find a profitable deviation for any pair a, b such that $\frac{1}{2} < a < b$. □

Given that we have assumed $a \leq b$, lemmas 1 and 3 imply that in a non-undercutting equilibrium, varieties have to satisfy $a \neq b$ and $0 \leq a \leq \frac{1}{2} \leq b \leq 1$.

5.1 Homogeneous costs

We first solve the case with homogeneous costs, i.e., $c_A = c_B = 0$. By plugging $c_B = 0$ into conditions (5), we find that, in an interior solution, ϕ^* is such that $\frac{F(\phi^*)}{f(\phi^*)} = \frac{3}{2}$ and $a + b = 1$, which means that both sellers locate symmetrically around the center. By Lemma 2, under the assumption of log-concavity, the first-order conditions are necessary and sufficient for an interior solution if it is feasible.

Lemma 2 also guarantees that under the assumption of log-concavity, there is a unique value of ϕ^* for which the first-order conditions of the two sellers are equal to zero, i.e., $\frac{F(\phi^*)}{f(\phi^*)} = \frac{3}{2}$. Consequently, in an interior solution, $a = \frac{1}{2} - \frac{\phi^*}{2}$ and $b = \frac{1}{2} + \frac{\phi^*}{2}$. Moreover, by lemma 3, an interior solution must satisfy $0 < a \leq \frac{1}{2} \leq b < 1$, which, given the previous interior values of a and b, can be written as $0 < \phi^* < 1$. The log-concavity of $F(\phi)$ implies that this condition is equivalent to $\frac{F(0)}{f(0)} \leq \frac{3}{2} < \frac{F(1)}{f(1)}$. If these two inequalities are not satisfied, corner solutions can arise. Given the above lemmas, there are four types of non-interior equilibrium candidates: $\{a = 0, b = 1\}$, $\{a = b\}$, $\{a \in (0, \frac{1}{2}], b = 1\}$ and $\{a = 0, b \in [\frac{1}{2}, 1)\}$. By Lemma 1, an equilibrium of the type $a = b$ cannot arise because it never satifies the non-undercutting conditions. For the rest of equilibrium candidates, we check first if the the first-order conditions are satisfied (we will check if the surviving candidates satisfy the non-undercutting conditions afterwards). For an equilibrium with $\{a = 0, b = 1\}$, it is necessary that $\frac{\partial \pi_A}{\partial a}(0,1) \leq 0 \leq \frac{\partial \pi_B}{\partial b}(0,1)$, which, by substituting $c_B = 0$ in (4), is equivalent to $\frac{F(1)}{f(1)} > \frac{3}{2}$. Similarly, one can see that the conditions on the two sellers' profits are incompatible in equilibria of the type $\{a \in (0, \frac{1}{2}], b = 1\}$ (which requires $\frac{\partial \pi_A}{\partial a}(a,1) = 0 \leq \frac{\partial \pi_B}{\partial b}(a,1)$) and $\{a = 0, b \in [\frac{1}{2}, 1)\}$ (which requires $\frac{\partial \pi_A}{\partial a}(0,b) \leq 0 = \frac{\partial \pi_B}{\partial b}(0,b)$). Then, we end up with the following pair of equilibrium candidates, for which we have to verify whether the non-undercutting conditions in Lemma 1 are satisfied:

- $a = \frac{1}{2} - \frac{\phi^*}{2}$ and $b = \frac{1}{2} + \frac{\phi^*}{2}$ if $\frac{f(1)}{F(1)} < \frac{2}{3} \leq \frac{f(0)}{F(0)}$.
- $a = 0$ and $b = 1$ if $\frac{2}{3} \leq \frac{f(1)}{F(1)}$.

The non-undercutting conditions are always satisfied for $\{a = 0, b = 1\}$. Finally, in the case of an interior solution, we can plug in the above expressions into the conditions in Lemma 1 to see that undercutting is not profitable if and only if $\phi^* \geq \frac{1}{2}$. The following proposition summarizes these results.

Proposition 1. *If $F(\phi)$ is log-concave, then a subgame perfect equilibrium in pure strategies exists if and only if $\frac{2}{3} \leq \frac{f(\frac{1}{2})}{F(\frac{1}{2})}$. At the variety stage of such an equilibrium:*

- $a = \frac{1}{2} - \frac{\phi^*}{2}$ *and* $b = \frac{1}{2} + \frac{\phi^*}{2}$ *if* $\frac{f(1)}{F(1)} < \frac{2}{3} \leq \frac{f(\frac{1}{2})}{F(\frac{1}{2})}$*, where* ϕ^* *is implicitly defined by* $\frac{f(\phi^*)}{F(\phi^*)} = \frac{2}{3}$.
- $a = 0$ *and* $b = 1$ *if* $\frac{2}{3} \leq \frac{f(1)}{F(1)}$.

Note that with symmetric costs, both sellers make zero profits in the competition for price-oriented buyers. In contrast, the prices that arise in the continuation subgame, i.e., $p_A = p_B = 1$, induce strictly positive profits from non-price-oriented buyers. Consequently, sellers experience a trade-off. On the one hand, they are tempted to "steal" some (positive-profit) non-price-oriented buyers from their competitor by getting closer to her. By doing so, however, they reduce the total amount of non-price-oriented buyers. If the latter effect is great enough, then an equilibrium in pure strategies exists in the variety game.

The existence of equilibrium contrasts with the results in D'Aspremont et al. (1979).[17] As they show, non-undercutting is only satisfied if the two sellers are sufficiently far

[17]Note that the presence of price-motivated buyers alone is insufficient to drive the existence of an equilibrium in pure strategies. The discounts are also necessary.

from each other.[18] With linear transportation costs, an equilibrium fails to exist in the Hotelling model because marginal reallocations in the directions of the best responses lead the sellers into the region where the non-undercutting conditions are violated. However, increasing the share of price-oriented buyers reduces sellers' incentives to approach each other. This makes possible the existence of an equilibrium in pure strategies, if it fully eliminates the two sellers' incentives to enter into the region where undercutting is profitable. In other words, an equilibrium can exist if the variety at which the sellers are indifferent between "stealing" buyers from her competitor and reducing the total amount of non-price-oriented buyers, takes place in the region where undercutting is not profitable. Such incentives are eliminated if the amount of price-oriented buyers increases sufficiently rapidly when sellers come closer to each other. Log-concavity imposes that buyers' transition rate to the price-oriented state is decreasing in $b - a$, which is insufficient for the existence of an equilibrium. In addition to log-concavity, the existence of an equilibrium in pure strategies requires the transition rate to be sufficiently large at the edge of the undercutting region.

If the previous condition is not satisfied, the possibility of increasing the share of non-price-oriented buyers is not dissuasive enough. Consequently, sellers have incentives to locate in the undercutting profitable regions, and an equilibrium in pure strategies does not exist, because cycles arise as in D'Aspremont et al. (1979). Note that a large mass of buyers with a ϕ value close to 0 means that many individuals behave as in a standard Hotelling model. Consequently, there would be no subgame perfect equilibrium in pure strategies as in the original game.

It is well known that price sensitivity can be affected by socioeconomic aspects as well as by culture; see Bijmolt et al. (2005) and Hoch et al. (1995). In our model, these differences are mediated by ϕ, which may be interpreted as the degree to which two differentiated goods can be seen as perfect substitutes by an individual. The distribution of ϕ is also subject to major changes, depending on the type of product. We consider changes in the distribution of ϕ in the following remark:

Remark 1. *When an equilibrium in pure strategies exists, distributional changes of ϕ that satisfy the monotone likelihood ratio property increase product differentiation.*

This remark arises directly from the expressions in Proposition 1. Shifting the entire distribution of ϕ according to the monotone likelihood ratio property implies an increase in the ratio $\frac{f(\phi)}{F(\phi)}$. Consequently, it would be more likely that the condition for a maximal differentiation equilibrium is satisfied at the same time that the distance increases between the two sellers in an interior solution. Thus, in equilibrium, product differentiation

[18]Symmetric equilibria also arise in Economides (1986a) by imposing non-linear and non-convex transportation costs.

increases by increasing the rate at which buyers become price-oriented for all $b-a$. Note that a necessary condition for a distributional change that satisfies the monotone likelihood ratio property is stochastic dominance. Consequently, an increase in the number of price-oriented buyers at any $b-a$ is a necessary but insufficient condition to increase product differentiation. Finally, a distributional change of ϕ that satisfies the monotone likelihood ratio property also facilitates the existence of a subgame perfect equilibrium.

5.2 Cost heterogeneity

Now, we consider the case in which $c_B > 0$. Proceeding as in the previous case, we can see that an interior solution can arise if and only if there is a pair of varieties satisfying the conditions in lemmas 1 and 3, such that:

$$a^* = \tfrac{3\sqrt{4c_B+1}-1-2c_B}{4} - \tfrac{\phi^*}{2} \quad \text{and} \quad b^* = \tfrac{3\sqrt{4c_B+1}-1-2c_B}{4} + \tfrac{\phi^*}{2}$$

where ϕ^* is such that $\frac{F(\phi^*)}{f(\phi^*)} = \frac{9-3\sqrt{4c_B+1}}{4}$.

The interior solution will arise if and only if it is feasible and satisfies the non-undercutting conditions. By taking into account these conditions, it turns out that an interior solution, i.e., $0 < a < \frac{1}{2}$ and $\frac{1}{2} < b < 1$, can only arise if the level of cost asymmetry is sufficiently small, i.e., $c_B \leq \frac{6}{25}$.[19] Otherwise, either corner solutions or no equilibrium in pure strategies arise. As in the symmetric cost case, no equilibrium in pure strategies exists if undercutting incentives arise. In contrast to the symmetric cost case, the corner solution can now be of two types, one in which only the higher-cost seller selects an extreme variety and the other in which the two sellers select extreme varieties. Let us define $\Psi(x) = \frac{F(x)}{f(x)}$ to characterize the equilibrium with heterogeneous marginal costs. For the sake of compactness, in the following proposition, we only characterize the equilibrium in the case in which the necessary condition for an interior solution is satisfied, i.e., $c_B \leq \frac{6}{25}$ (see the appendix for a proof).[20]

Proposition 2. *If $F(\phi)$ is log-concave and $c_B \leq \frac{6}{25}$, a subgame perfect equilibrium in pure strategies exists if and only if $\Psi(\frac{5\sqrt{4c_B+1}-3-6c_B}{4}) \leq \frac{9-3\sqrt{4c_B+1}}{4}$. At the variety stage of such an equilibrium:*

- *$a = \frac{3\sqrt{4c_B+1}-1-2c_B}{4} - \frac{\phi^*}{2}$ and $b = \frac{3\sqrt{4c_B+1}-1-2c_B}{4} + \frac{\phi^*}{2}$ if $\Psi(\frac{5\sqrt{4c_B+1}-3-6c_B}{4}) \leq \Psi(\phi^*) \leq \Psi(\frac{5+2c_B-3\sqrt{4c_B+1}}{2})$, where ϕ^* is implicitly defined by $\Psi(\phi^*) = \frac{9-3\sqrt{4c_B+1}}{4}$*
- *$a \in (0, \frac{3\sqrt{4c_B+1}-3-2c_B}{2}]$ and $b = 1$ if $\Psi(\frac{5+2c_B-3\sqrt{4c_B+1}}{2}) \leq \frac{9-3\sqrt{4c_B+1}}{4}$ and $\Psi(1) > \frac{3+c_B}{2} - \frac{9c_B}{3+c_B}$, with equilibrium platform of party A implicitly defined by $\Psi(1-a) = \frac{3+a+c_B}{2} - \frac{9c_B}{3+a+c_B}$*

[19] The absolute threshold $\frac{6}{25}$ arises from the assumption $c_A = 0$.

[20] We characterize the more general case $0 < c_B \leq 2$ in the proof of proposition 2.

- $a = 0$ *and* $b = 1$ *if* $\Psi(1) \leq \frac{3+c_B}{2} - \frac{9c_B}{3+c_B}$

The key difference with respect to the homogeneous case is that now seller A has a lower marginal cost than B, thus she makes positive profits from charging a price equal to seller's B marginal cost. Consequently, seller A has higher incentives to draw closer than seller B, as she is less harmed by the presence of price-oriented buyers. As before, seller B makes zero profits from price-oriented buyers, so she will be more reluctant to have a similar variety to the one of seller A.

There are three possible types of equilibrium. In the first type, both players choose a non-extreme product. However, seller A has a more central variety than seller B. This equilibrium can only emerge if the cost advantage of seller A is sufficiently small, i.e., $c_B \leq \frac{6}{25}$. An equilibrium in which both sellers offer non-extreme products requires two additional conditions from the distribution function of ϕ. On the one hand, the transition rate of buyers who become price-oriented if sellers marginally move towards each other must be great enough to prevent sellers from locating at the undercutting region. On the other hand, it is also necessary that the rate of buyers who become price-oriented is small enough to prevent the disadvantaged seller B from preferring an extreme product. Hence, this equilibrium requires a transition rate of buyers from a non-price-oriented to price-oriented state to be bounded from above and below.

In the other two types of equilibrium, seller B always chooses an extreme variety. In these equilibria, the transition of buyers who become price oriented at any point in the non-undercutting region makes seller B willing to locate as far as possible from seller A. In other words, seller B prefers to minimize the amount of price-oriented buyers rather than try to "steal" non-price-oriented buyers from seller A. Seller A also chooses an extreme product if the buyers' transition to the price-oriented state is large even at maximal differentiation, i.e., if $\frac{f(1)}{F(1)}$ is large enough. Otherwise, A chooses an interior variety.

Although now it cannot be as easily seen as above, the changes in the distribution of ϕ follow the same logic as in the homogeneous case and are skipped. A new result emerges by looking at equilibrium prices. In particular, we find that it may be the case that the seller with a lower marginal cost fixes a higher price than the one with a lower marginal cost. This is highlighted in the following remark (see the appendix for a proof).

Remark 2. *The seller with a lower marginal cost may fix a higher price than the seller with a higher marginal cost. This is always the case if, in equilibrium, sellers choose interior varieties.*

This is a counterintuitive result that contrasts with the general wisdom in which one may expect a lower cost seller to exploit its advanatge by fixing a lower price. When serving non-price oriented buyers, seller A does not exploit its cost advanatage to sell the good at a lower price but to produce a more central variety of the product, i.e., $a+b>1$. By looking at the equilibrium prices, we can see that $p_A^* \geq p_B^* \iff a+b \geq 1+\frac{c_B}{2}$. This is always true when the two sellers choose interior varieties, i.e., $a+b=\frac{3\sqrt{4c_B+1}-1-2c_B}{2}$, while it depends on the variety of seller A when only seller A chooses an interior variety. Thus, the higher share of non-price-oriented buyers who prefer product A allow seller A to fix a higher price than her competitor. Recall that, as explained above, the dominant position enjoyed by seller A emerges as a consequence of her ability to capture the whole market of price-oriented buyers. This implies that cost asymmetry alone is insufficient to generate this result. It also requires the presence of price-oriented buyers.[21]

Despite its counterintuitive nature, the possibility that a lower-cost firm charges a higher price than a higher-cost firm is not theoretically new. When the demand of the low-cost firm is sufficiently less elastic than that of the high-cost firm, the profit-maximizing mark-up rule studied in textbook models of monopolistic competition also implies this result. A nice aspect of our model is that it provides an endogenous rationale that justifies why the demand of a lower-cost seller may be less elastic. The elasticity of the demand of seller A can be calculated as $\epsilon_A = \frac{-p_A}{a+b+p_B-p_A}$. As we have seen with equal costs, product varieties satisfy $a+b=1$, while in the asymmetric case $a+b>1$. This implies that product varieties chosen in the asymmetric cost case reduce the elasticity of the demand of seller A. A similar argument shows that the opposite is true for seller B. Although we present a model of price competition, our results can also be explained in the following terms. The (dis)advantage experienced by the lower- (higher-) cost seller in the segment of price-oriented buyers induces her to choose a more (less) central product variety with a less (more) elastic demand. Consequently, in any interior solution, the lower-cost seller charges a higher price than the higher-cost seller.

6 Discussion

The present article uses the notion of semiorder preferences to endogenously divide a population of buyers into two differently behaved groups. The main idea is clear and intuitive: individuals are heterogeneous in the existing difference on an attribute that they qualify as (ir)relevant. This idea may be applied to a wide variety of contexts and consequently our contribution should not be restricted to the study of product differenti-

[21] To see this, we can change our model by assuming that price-oriented buyers do not purchase any product. Consequently, even in the presence of asymmetric marginal costs, both sellers make zero profits from price-oriented buyers. Proceeding as above to solve the model, one sees that in that case, the lower-cost seller never enjoys a favorable position in the differentiation space, and consequently, the seller never fixes a higher price.

ation. For instance, studies in Political Science often take into account models of spatial competition similar to the Hotelling model in which our proposal can also be applied. In an ongoing research, Balart et al. (2017) apply it to study the impact of technology change on electoral outcomes and show how it can explain several trends observed in the US politics. The tractability of the model together with its intuitive motivation may encourage other researchers to use it in their studies.

Focusing on the topic of product differentiation, we contribute to study the implications of consumer behavior on sellers' strategies. Introducing the above behavioral consideration allows to endogenize the presence of price-oriented buyers. Although price-oriented buyers do not care about product differences, we show that their existence notably affects how firms should design their products. In equilibrium, the degree of product differentiation depends on the sensitivity of the population to product differences. The rate at which the population becomes price-oriented when the two varieties become more similar, positively affects the equilibrium level of product differentiation. In the presence of asymmetric production costs, the lower-cost seller can use her advantage to gain market power in the horizontal dimension. Consequently, a subgame perfect equilibrium with potentially asymmetric varieties can arise in which, counterintuitively, the lower-cost producer charges a higher price.

The discounts introduced in our model might also be interpreted as a form of vertical differentiation. That interpretation would liken our model to spatial models that combine horizontal and vertical differentiation; see Brekke et al. (2006 and 2010), Champsaur and Rochet (1989), Economides (1986b and 1989), Irmen and Thisse (1998) or Neven and Thisse (1987), among others. If we interpret the discount as a vertical attribute in the sense that all buyers prefer a higher level of discount, the main difference with respect to the other models that combine vertical and horizontal differentiation is that in our case, the two attributes are not substitutes. The vertical attribute would be dominated by the horizontal one. While the substitutability assumption is valid in many cases, there might be others in which the vertical attribute might be dominated for (some) consumers, e.g., the hard disk memory of a computer or the gas emission of a car. This type of information is usually not the main concern of potential buyers, but it is publicly available in the specifications of the product and may determine the final decision in case of indifference in the dominant attribute.

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7 Appendix

7.1 Proof of proposition 2

Given that we have retained the case for $c_B \leq 2$ and considering the constraints $0 \leq a \leq \frac{1}{2} \leq b \leq 1$, the interior solution is feasible if and only if $\frac{3\sqrt{4c_B+1}-3-2c_B}{2} < \phi^* < \frac{5+2c_B-3\sqrt{4c_B+1}}{2}$. Notice that the previous interval is non-empty if $c_B \leq \frac{5-3\sqrt{2}}{2}$. By plugging a^* and b^* in the non-undercutting conditions, we can see that they are not binding for seller A, taken into account the previous constraints on ϕ^* and c_B. However, the non-undercutting condition for seller B requires $\frac{5\sqrt{4c_B+1}-3-6c_B}{4} < \phi^*$ which is a more restrictive condition than $\frac{3\sqrt{4c_B+1}-3-c_B}{2} < \phi^*$ given $c_B \leq \frac{5-3\sqrt{2}}{2}$. Thus, the interior solution arises if and only if $\frac{5\sqrt{4c_B+1}-3-6c_B}{4} \leq \phi^* < \frac{5+2c_B-3\sqrt{4c_B+1}}{2}$, or equivalently for $\frac{F[\frac{1}{4}(5\sqrt{4c_B+1}-3-6c_B)]}{f[\frac{1}{4}(5\sqrt{4c_B+1}-3-6c_B)]} \leq \frac{9-3\sqrt{4c_B+1}}{4} < \frac{F[\frac{1}{2}(5+2c_B-3\sqrt{4c_B+1})]}{f[\frac{1}{2}(5+2c_B-3\sqrt{4c_B+1})]}$, which is a non-empty interval provided that $c_B \leq \frac{6}{25}$. Thus $c_B \leq \frac{6}{25}$ is a necessary condition for the existence of an equilibrium in which the two sellers select interior varieties.

Besides the interior solution and given the constraints on a and b the rest of the equilibrium candidates are: $\{a = 0, b = 1\}$, $\{a = b = \frac{1}{2}\}$, $\{a \in (0, \frac{1}{2}], b = 1\}$, $\{a \in (0, \frac{1}{2}], b = 1\}$ and $\{a = 0, b \in [\frac{1}{2}, 1)\}$.

We provide the necessary and sufficient conditions to guarantee the existence of each type of solution which allows us to proof Proposition 2 as well as to characterize equilibrium for any value of c_B in the interval $c_B \in [0, 2]$.

- $\{a = b\}$.

  By taking into account the non-undercutting conditions in Lemma 1 we can see that this can never be an equilibrium.

- $\{a \in (0, \frac{1}{2}], b = 1\}$

  By lemma 2, if the non-undercutting conditions were satisfied, the necessary and sufficient conditions for an equilibrium of the type $\{a \in (0, \frac{1}{2}], b = 1\}$ are:

$$\frac{\partial \pi_A}{\partial a}(a, 1) = 0 \iff \frac{F(1-a)}{f(1-a)} = \frac{3+a+c_B}{2} - \frac{9c_B}{3+a+c_B}$$

$$\frac{\partial \pi_B}{\partial b}(a, 1) \geq 0 \iff \frac{3-a-c_B}{2} \geq \frac{F(1-a)}{f(1-a)}$$

  which requires $0 \leq \frac{3+a+c_B}{2} - \frac{9c_B}{3+a+c_B} \leq \frac{3-a-c_B}{2}$. These inequalities are satisified if and only if $a \in [0, \frac{3\sqrt{4c_B+1}-3-2c_B}{2}]$ and $c_B \in (0, 3(2-\sqrt{3})]$ or $a \in [3\sqrt{2c_B} - 3 - c_B, \frac{3\sqrt{4c_B+1}-3-2c_B}{2}]$ and $c_B \in (3(2-\sqrt{3}), 2]$. Now taking into account undercutting conditions in lemma 1, undercutting incentives never arise for seller A. However, this is not the case for seller B, for which the non-undercutting condition requires $a \leq 15 - 6\sqrt{6-c_B} - c_B$ for $c_B \leq 2$, which is more restrictive than $a \leq \frac{3\sqrt{4c_B+1}-3-2c_B}{2}$ if and only if $c_B > \frac{6}{25}$. Combining all these constraints on a and c_B we obtain that:

- If $0 < c_B \leq \frac{6}{25}$ and $a \leq \frac{3\sqrt{4c_B+1}-3-2c_B}{2}$. Equilibrium of the type $\{a \in (0, \frac{1}{2}], b = 1\}$ exists if and only if $\frac{\partial \pi_A}{\partial a}(\frac{3\sqrt{4c_B+1}-3-2c_B}{2}, 1) \leq 0$ and $\frac{\partial \pi_A}{\partial A}(0,1) > 0$. The former can be written as $\frac{F(\frac{1}{2}(5-3\sqrt{4c_B+1}+2c_B))}{f(\frac{1}{2}(5-3\sqrt{4c_B+1}+2c_B))} \leq \frac{9-3\sqrt{4c_B+1}}{4}$ while the latter can be written as $\frac{F(1)}{f(1)} > \frac{3+c_B}{2} - \frac{9c_B}{3+c_B}$.
- If $\frac{6}{25} < c_B \leq 3(2-\sqrt{3})$ and $a \leq 15 - 6\sqrt{6-c_B} - c_B$. Equilibrium of the type $\{a \in (0, \frac{1}{2}], b = 1\}$ exists if and only if $\frac{\partial \pi_A}{\partial a}(15 - 6\sqrt{6-c_B} - c_B, 1) \leq 0$ and $\frac{\partial \pi_A}{\partial a}(0,1) > 0$. The former can be written as $\frac{F(6\sqrt{6-c_B}+c_B-14)}{f(6\sqrt{6-c_B}+c_B-14)} \leq \frac{18-6\sqrt{6-c_B}}{2} - \frac{9c_B}{18-6\sqrt{6-c_B}}$ while the latter can be written as $\frac{F(1)}{f(1)} > \frac{3+c_B}{2} - \frac{9c_B}{3+c_B}$.
- If $3(2-\sqrt{3}) < c_B \leq 2$ and $3\sqrt{2c_B} - 3 - c_B < a \leq 15 - 6\sqrt{6-c_B} - c_B$. Equilibrium of the type $\{a \in (0, \frac{1}{2}], b = 1\}$ exists if and only if $\frac{\partial \pi_A}{\partial a}(15 - 6\sqrt{6-c_B} - c_B, 1) \leq 0$ and $\frac{\partial \pi_A}{\partial a}(3\sqrt{2c_B} - 3 - c_B, 1) \geq 0$. The former can be written as $\frac{F(6\sqrt{6-c_B}+c_B-14)}{f(6\sqrt{6-c_B}+c_B-14)} \leq \frac{18-6\sqrt{6-c_B}}{2} - \frac{9c_B}{18-6\sqrt{6-c_B}}$ while the latter can be written as $\frac{F(4+c_B-3\sqrt{2c_B})}{f(4+c_B-3\sqrt{2c_B})} \geq 0$.

- $\{a = 0, b \in [\frac{1}{2}, 1)\}$ By lemma 2, necessary conditions for an equilibrium with $\{a = 0, b \in [\frac{1}{2}, 1)\}$ are:

$$\frac{\partial \pi_A}{\partial a}(0,b) \leq 0 \iff \frac{F(b)}{f(b)} \leq \frac{2+b+c_B}{2} - \frac{9c_B}{2+b+c_B}$$

$$\frac{\partial \pi_B}{\partial b}(0,b) = 0 \iff \frac{4-b-c_B}{2} = \frac{F(b)}{f(b)}$$

which requires $\frac{4-b-c_B}{2} \leq \frac{2+b+c_B}{2} - \frac{9c_B}{2+b+c_B}$ which is never true for $c_B \leq 2$.

- $\{a = 0 \text{ and } b = 1\}$

Using the conditions in lemma 1, we see that there are no under-cutting incentives for any seller when $a = 0$ and $b = 1$. Consequently, by lemma 2, the necessary and sufficient conditions for an equilibrium with $a = 0$ and $b = 1$ are:

$$\frac{\partial \pi_A}{\partial a}(0,1) \leq 0 \iff \frac{F(1)}{f(1)} \leq \frac{3+c_B}{2} - \frac{9c_B}{3+c_B}$$

$$\frac{\partial \pi_B}{\partial b}(0,1) \geq 0 \iff \frac{3-c_B}{2} \geq \frac{F(1)}{f(1)}$$

The condition for seller A is more restrictive for all $c_B \leq 2$ and consequently it is the necessary and sufficient one for this type of equilibrium. Note that a necessary condition for $\frac{\partial \pi_A}{\partial a}(0,1) \leq 0$ is $\frac{3+c_B}{2} - \frac{9c_B}{3+c_B} \geq 0$, which is always satisfied for $c_B \leq 2$.

Note that the intervals for which we have found the equilibria in pure strategies are mutually exclusive, hence each equilibrium is unique in its interval. Given that we have considered all possible type of pure strategy Nash equilibrium, it implies the non-existence of a Nash equilibrium in pure strategies outside any of the previous regions. For $c_B \leq \frac{6}{25}$, this means that there is no equilibrium in pure strategies for $\frac{9-3\sqrt{4c_B+1}}{4} < \frac{F[\frac{1}{4}(5\sqrt{4c_B+1}-3-6c_B)]}{f[\frac{1}{4}(5\sqrt{4c_B+1}-3-6c_B)]}$.

7.2 Proof of Remark 2

Using pure strategy equilibrium prices, i.e., $p_A^* = \frac{2+a+b+c_B}{3}$ and $p_B^* = \frac{4-a-b+2c_B}{3}$, we can see that $p_A^* \geq p_B^* \iff a+b \geq 1 + \frac{c_B}{2}$. We also know that in an interior solution $a+b = \frac{3\sqrt{4c_B+1}-1-2c_B}{2}$. Combining the two expressions we see that given an interior solution at the variety stage, $p_A^* \geq p_B^* \iff 1 + c_B \leq \sqrt{4c_B+1}$, which always holds for $c_B \leq 2$. Thus, $p_A^* > p_B^*$ if interior varieties arise in equilibrium. For the other types of equilibrium, we can immediately see that $p_A^* \geq p_B^*$ never arises for an equilibrium of the type $a = 0$ and $b = 1$. Finally, for an equilibrium of the type $\{a \in (0, \frac{1}{2}], b = 1\}$, $p_A^* \geq p_B^*$ if and only if in equilibrium $a \geq \frac{c_B}{2}$.